

Diffusion Model for Fluidized-Bed Drying

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Abstract □ A sucrose-lactose-starch granulation was used to study particulate motion and attrition in a fluid bed dryer. There is some classification of material in the dryer as drying proceeds; fine particles are dried faster and become less dense, and the less dry but denser large particles show some (although not great) accumulation tendencies in the lower central area. Unlike countercurrent rotary drying, fluid bed drying cannot be accounted for by water diffusion inside the granule as the rate-limiting step. In its place, a model of external water vapor diffusion is proposed and is supported by vapor-concentration curves and by the linear dependence of the rate constants on the linear air velocities. The dried granulation exhibits the same trend as does countercurrent dried material in that larger particles have higher moisture contents than do smaller particles. Quantitative relationships between content of moisture and size were developed and are supported by experimental data. The granulation, upon storage, does not equilibrate, indicating that this type of water distribution is a problem in batch process granulations as well as in the earlier reported case of granulations for continuous production.

Keyphrases □ Drying—fluid bed dryer, diffusion model, sucrose-lactose-starch granulation, rate-limiting step, relationships between moisture and size □ Diffusion—model for fluidized-bed drying, moisture and size relationships, rate-limiting step □ Fluid bed drying—diffusion model, moisture and size relationships, rate-limiting step, equations

Drying of granulations has traditionally been performed by tray drying, and the factors involved have been classified as a dynamic balance between the rate of moisture movement from inside the solid to the surface and the evaporation of moisture from the surface (1, 2). It has been shown (3) that the drying rate is controlled by the film thickness of the vapor above the bed and that there is no difference fundamentally between the heat balances involved in fluidized drying and those involved in tray drying (4). In countercurrent rotary dryers, the rate-determining step is diffusion of moisture in the granule (5). An investigation of the basic mass transfer balance in a fluidized bed at fluidizing air velocities was the primary purpose of the study reported here.

The general phenomena involved in fluid bed drying are described in various texts; the text by Vanecek *et al.* (6) will be referred to frequently. Fluidization occurs between an incipient fluidizing velocity and is limited at the higher end by an entrainment velocity beyond which conveyance takes place. The linear velocity of the air in the bed is different from that in the ducts. The velocity between the particles is referred to as interstitial velocity; the velocity in the free cross section, *i.e.*, in the ducts, is denoted as superficial velocity.

One purpose of this study was to determine whether particulate fluidization or aggregative fluidization takes place. In the former case, the bed is uniform

throughout; in the latter case, it is not. The solids motion in the bed is somehow controlled by the fact that bubbles are created at the distributor. Not only the bubbles but also their wakes contribute to the solids motion (7). Bubbles do not mix in a horizontal plane, but they coalesce whenever a bubble is less than one bubble diameter behind another in a vertical path (8). Therefore, bubble size is less above the distributor than higher up in the bed.

Several "freezing" techniques were described in investigations of mixing, such as the one by Abrahams and Resnick (9). Essentially, it is shown that mixing will occur beyond the incipient fluidization velocity. The actual mixing patterns in a fluid bed dryer were described by Schugerl (10).

In a mixture of two particle sizes, it was shown that when one fraction (f_1) is abundant there is a tendency for accumulation of the coarse fraction above the distributor (11).

The temperature of the material being dried is frequently uniform throughout the bed, except for a small range immediately above the distributor (12). It is not possible to obtain a countercurrent arrangement of the streams of the material and drying medium in a fluid bed dryer (6). Therefore, one would not expect that the drying mechanism reported in countercurrent drying (5) would apply to fluid bed drying.

Although attempts to quantitate the stream lines in a fluidized bed have been made (6), no previous studies reported particle-size distributions and classifications (or lack of classifications) in fluid bed dryers. It was observed in continuous rotary drying (5) that the moisture content of various screen fractions of a continuously dried granulation differs in the sense that the coarse particles contain more moisture than the fines. It was speculated that this fact would be of no consequence in batch operations, since the particles would, presumably, equilibrate. This latter point was investigated in this study.

EXPERIMENTAL

Granulations were made of the following formula: 17.55 kg of lactose USP, 11.43 kg of sucrose (containing 3% starch), 4.20 kg of starch USP, 0.0093 kg of yellow FD&C No. 6, and 0.0045 kg of red FD&C No. 2. The lactose, sucrose, and 4.05 kg of starch were mixed for 5 min in a mixer¹ at a speed of 140 rpm. A starch suspension was made of 150 g of starch in 300 ml of water. The dyes were dissolved in 2700 ml of water and heated to boiling; then the starch suspension was added to make approximately 3 liters of dye starch

¹ Littleford-Lodge model FM-100, 3.3 ft³ capacity, Littleford Brothers, Inc., Cincinnati, OH 45202

paste. One-fourth of this paste was added to the powder blend in the mixer, and blending was carried out for 30 sec; the next one-fourth was then added and so on until all of the paste had been added.

The wet granules (5.0 kg) were screened through a No. 4 stainless steel hand screen and placed in the basket of a fluid bed dryer². The fluid bed dryer was equipped with a velocity meter³ of the Pitot-tube type with wet and dry bulb thermometers in inlet and outlet ducts. The position of the Pitot-tube was (as recommended by the manufacturer) positioned on linear portions of the duct work. The velocity could be varied by a conventional damper system. The ranges of velocity were such that no fluidization took place below 396 m (1300 ft)/min air velocity; above 823 m (2700 ft)/min air velocity, air conveyance would start. The range used was 396–815 m (1300–2675 ft)/min. The inlet temperature was 43°, and the exit temperature was 30°.

The heat capacity of the granulation was determined by calorimetric means. A Dewar flask, equipped with a rubber stopper and a Beckmann thermometer, was placed in a 20° constant-temperature bath along with 250 ml of heptane. When the temperature was equilibrated, the heptane was transferred to the Dewar flask. Then 100 g of granulation was heated to 30° by equilibrating it for 6 hr in a glass container in a 30° constant-temperature bath. It was then transferred to the Dewar flask, and the temperature increase was recorded. The calorimetric constant (80 cal/degree) was determined by water calibration. The heat capacity is 0.45 cal/g/degree of dry weight.

THEORETICAL

It will be assumed that the drying chamber is cylindrical (although in actuality it is slightly conical) with a cross section of A cm² and height of l cm. When fluidized, the bed has a void fraction (in the conventional sense) of ϵ and a solid fraction of $(1 - \epsilon)$. The free cross-sectional area (7) is then, on the average, $A\epsilon$. The granulation is polydisperse, containing fraction f_i of particles with "diameter" a_1, f_2 with "diameter" a_2 , and so on. The assumption is made that the particle is spherical.

In a small section of the cylinder of length dx , the total solids volume is $A dx(1 - \epsilon)$. The volume of particles of diameter a_i in the section is $A dx(1 - \epsilon)f_i$. There are, hence, $A dx(1 - \epsilon)f_i/(\pi a_i^3/6)$ particles of diameter a_i , each having a surface of πa_i^2 , so that the total solids area in the section of length dx is then given by:

$$O = A dx(1 - \epsilon) \sum_{i=1}^n f_i \frac{6}{a_i} \quad (\text{Eq. 1})$$

The drying, as shown in Fig. 1, is governed by diffusion of water vapor from the solid surface (where the gaseous concentration is q^* in grams of water per cubic centimeter of dry air) to the bulk of the air stream where the water content at position x is q_x in grams of water per cubic centimeter of dry air. The interface where the water vapor concentration has just dropped to q_x is h cm from the granule surface and is referred to here as the "film." By way of Fick's first law applied to the length of this film, when a linear concentration gradient is assumed (8), the rate of moisture given off at time t and position x from the solids to the air stream is:

$$\frac{D}{h} (q^* - q_x) A (1 - \epsilon) \sum_{i=1}^n (f_i 6/a_i) dx = \gamma (q^* - q_x) A dx \quad (\text{Eq. 2})$$

where:

$$\gamma = \frac{D}{h} (1 - \epsilon) \sum_{i=1}^n (f_i 6/a_i) \quad (\text{Eq. 3})$$

and D is the diffusion coefficient for the water vapor in the gas stream.

The total moisture content of the solids is denoted m_0 and m at times zero and t sec, respectively. The moisture loss rate of the total solids mass is obtained by integrating Eq. 2 between the limits zero and l , i.e.:

$$-\frac{dm}{dt} = \gamma A \int_0^l (q^* - q_x) dx \quad (\text{Eq. 4})$$

If the relative humidity ($100q_x/q^*$) gradient is constant, then:

$$q_x = q_0 + \frac{q_l - q_0}{l} x \quad (\text{Eq. 5})$$

where $100q_l/q^* = \text{RH}$ is the relative humidity of the exit air. The average water vapor concentration, q , is:

$$q = (q_l + q_0)/2 \quad (\text{Eq. 6})$$

Combining Eqs. 5 and 6 with Eq. 4 yields:

$$-\frac{dm}{dt} = \gamma A \{ (q^* - q_0)l - [\frac{1}{2}(q_l - q_0)/l] l^2 \} =$$

$$\gamma A l [q^* - \frac{1}{2}(q_l + q_0)] = \gamma A l (q^* - q) \quad (\text{Eq. 7})$$

When the system is considered to be a suspension with a solids volume of $lA(1 - \epsilon)$ and an air volume of $lA\epsilon$, the increase in average concentration of water vapor in the gas is the amount of water given up by the solids divided by the gas volume $[-(dm/dt)/(lA)]$ minus the amount lost in the gas stream, i.e., the difference between incoming and outgoing moisture quantities⁴ $[vA\epsilon(q_l - q_0)]$ divided by the gas volume, i.e.:

$$\frac{dq}{dt} = \frac{1}{lA} \frac{dm}{dt} - \frac{vA\epsilon}{lA} (q_l - q_0) \quad (\text{Eq. 8})$$

where v is the linear velocity of the air. Inserting Eqs. 7 and 6 into Eq. 8 now yields:

$$\frac{dq}{dt} = \frac{\gamma}{\epsilon} (q^* - q) - \frac{2v}{l} (q_l - q_0) = \alpha (q - \beta q^* - \sigma q_0) \quad (\text{Eq. 9a})$$

where:

$$\alpha = - \left(\frac{\gamma}{\epsilon} + \frac{2v}{l} \right) < 0 \quad (\text{Eq. 9b})$$

$$\beta = \frac{(\gamma/\epsilon)}{(\gamma/\epsilon) + (2v/l)} \quad (\text{Eq. 9c})$$

$$\sigma = \frac{(2v/l)}{(\gamma/\epsilon) + (2v/l)} \quad (\text{Eq. 9d})$$

It is noted that $\alpha + \sigma = 1$. Integration of Eq. 9a yields:

$$\ln (q - \beta q^* - \sigma q_0) = \alpha t + I \quad (\text{Eq. 10})$$

where I is an integration constant to be determined by initial conditions. Boundary conditions dictate that when t becomes infinite, the term under the logarithm sign approaches zero; i.e., since q then approaches q_0 :

$$q_0 - \beta q^* - \sigma q_0 = 0 \quad \beta = (1 - \sigma)q_0/q^* \quad (\text{Eq. 11a})$$

Inserting Eq. 11a into Eq. 10 then gives:

$$\ln (q - q_0) = \alpha t + I \quad (\text{Eq. 11b})$$

where I is an integration constant. The value of I can be obtained from initial conditions; here the inlet air has a moisture content of q_0 and a (saturated) content of q^* at the outlet, so that initially $q = (q_0 + q^*)/2$, which, inserted in Eq. 11b, yields:

$$\ln [(q_0 + q^*)/2 - q_0] = 0 + I \quad (\text{Eq. 11c})$$

so that $I = \ln [(q^* - q_0)/2]$. The complete equation may be written:

$$\ln [(q_l - q_0)/2 - q_0] = \alpha t + \ln [(q^* - q_0)/2] \quad (\text{Eq. 12a})$$

or:

$$\log (\text{RH} - \text{RH}_0) = (\alpha/2.3)t + \log (100 - \text{RH}_0) \quad (\text{Eq. 12b})$$

where $\alpha < 0$. Data plotted according to Eq. 12b (Fig. 2) show that this equation is obeyed at all air velocities tested. It is noted that:

$$-\alpha = \frac{\gamma}{\epsilon} + \frac{2v}{l} \quad (\text{Eq. 13})$$

so that the negative slopes in Fig. 2 multiplied by 2.3 should be linearly related to the linear velocity, as demonstrated in Fig. 3.

² Aeromatic AG model STR-5, Aeromatic Ltd., Basel, Switzerland.

³ Catalog No. 102AV, Dwyer Instruments Inc., Michigan City, IN 46360

⁴ It is noted that q_0 and RH_0 are equal to the values in the outlet at infinite time.

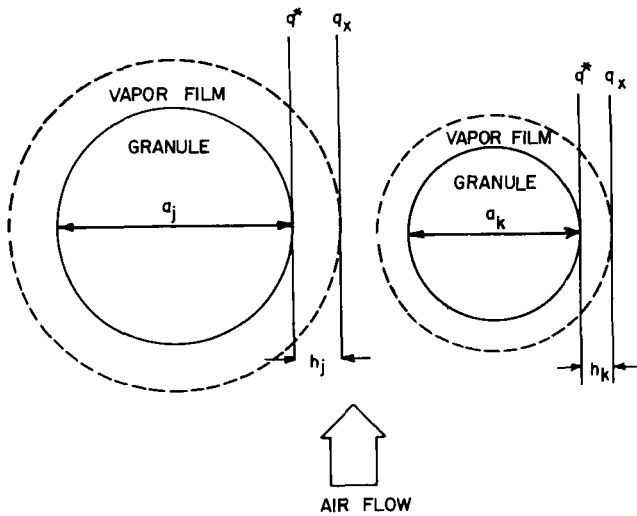


Figure 1—Particles of diameter a_j and a_k and film thicknesses h_j and h_k .

If it is assumed that all sieve fractions start with the same moisture content and level off at about 0.04 g of water/g of dry solid, then the moisture loss rates may be estimated from Table I. In particular, at any given time, according to Eqs. 3 and 7:

$$(dm_i/dt)_{t=t'} = D(1 - \epsilon)(6f_i/a_i h_i) A l (q^* - q_v) \quad (\text{Eq. 14})$$

Data from $t = 1.25$ min calculated from Table I are shown graphically in Fig. 4.

According to hydrodynamic theory (8), diffusion in turbulent flow occurs through a film of thickness h , which is related to the Reynolds number (Re) by an exponential relation, i.e., $h = (\omega) Re^{-(\zeta-1)}$. Here, $Re = v(a/\nu)$, where v is the velocity of the stream flowing past the body, a is the characteristic dimension of the body, and ν is the kinematic viscosity of the stream. Therefore, it follows that Eq. 14 will take the form:

$$\log[-(dm_i/dt)_{t=t'}] = (\zeta - 2) \log a_i + \log \left[D(1 - \epsilon)(6f_i) A l (q^* - q_v) \left(\frac{1}{\omega} \right) \right] + (\zeta - 1) \log (v/\nu) \quad (\text{Eq. 15})$$

Figure 4 shows that Eq. 15 is obeyed, the value of $(\zeta - 2)$ being -0.40 . Hence, $\zeta = 1.6$ and $h = (\omega) Re^{-0.6}$. This result is close to the usual hydrodynamic postulate that the sorbed "film" of fluid (or gas) is inversely proportional to the square root of the velocity (13). This should not be construed as implying quantitative proof of the proposed model, but the adherence to Eq. 15 lends credence to the proposed view.

RESULTS AND DISCUSSION

The granulation from a 15-min drying experiment, conducted in the manner described, was sampled from the top (2–3 cm deep) in a zone at the center in a circle with radius 7.6 cm (3 in.), then in a zone 7.6–15.4 cm (3–6 in.) from the center, and then in a zone more than 15.4 cm (6 in.) from the center. The samples were subjected to a sieve analysis through a nest of standard U.S. screens, and the various screen fractions were subjected to a Karl Fischer moisture analysis (Table II).

The screen analysis is fairly independent of the zone, and the moisture analysis shows that the coarse material is somewhat moister than the fine material. Apparently, there is a slight tendency for a larger proportion of the coarse fraction to be positioned at the center of the distributor; this so-called immobile zone has a minimal effect on the general particle distribution. Initially, the wet granules (prior to drying) contain 97.5% (w/w) of granules coarser than 60 mesh; these granules contain 9.5% moisture. The 2.5% material finer than 60 mesh is slightly drier (6.4%).

The next experiment utilized a 5-min drying time. The dried granulation was mixed in a large plastic bag, and the material was subjected to sieve analysis. The experiment was then repeated for 15, 30, and 45 min (Table III). Table III shows that the particle-

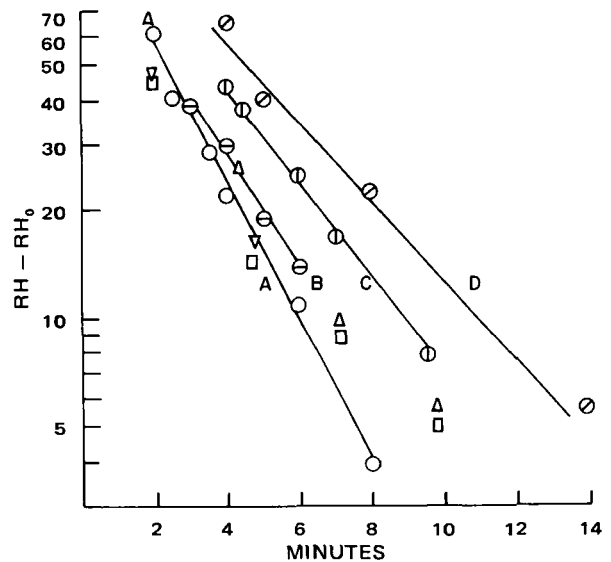


Figure 2—Relative humidity of outlet air minus relative humidity of ambient air ($RH - RH_0$) plotted in semilogarithmic fashion versus time. Least-squares fit slopes are: A (470 cm/sec), -0.186 min^{-1} ; B (460 cm/sec), -0.155 min^{-1} ; C (407 cm/sec), -0.130 min^{-1} ; and D (274 cm/sec), -0.114 min^{-1} . Key: Δ , 37° (460 cm/sec); ∇ , 30° (460 cm/sec); and \square , 25° (460 cm/sec).

size distribution stays fairly constant throughout the drying and that the formula is resistant to the possible abrasion in the fluidizing motion.

Moisture assays by Karl Fischer titration were performed on various fractions from the experiments (Table I). Table I illustrates the increased drying time necessary to bring the larger granules to moisture levels comparable to those achieved rapidly by the smaller granules. A sample was dried for 2.5 min and blended and screened as described. Samples of the various fractions were assayed for moisture; the remainder of the batch was reblended and placed in a plastic bag in a sealed fiber drum for 2 days. Screening, sampling, assay, blending, and restorage were repeated twice. Inspection of Table IV shows that equilibration of the tested formula does not occur subsequent to drying and that the previous observation (5) that the uneven distribution of moisture could be a problem in continuous processes is apparently also true for batch processing.

A series of runs was then performed in which the linear velocity of the air was varied; the temperatures of the incoming air and the temperatures of the exit air were monitored to permit calculation of the relative humidities of inlet and outlet streams at all times (Fig. 2).

The process that dominates continuous rotary drying, as found by Pitkin and Carstensen (5), dictates (as detailed in the Appen-

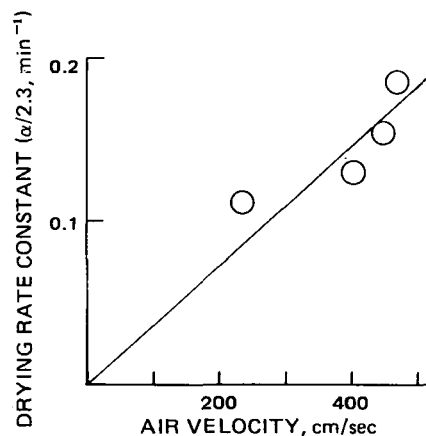


Figure 3—Negative slopes from Fig. 2 plotted versus linear velocities.

Table I—Karl Fischer Moisture Contents of Various Mesh Cuts as a Function of Drying Time

Minutes	20 Mesh		40 Mesh		60 Mesh		80 Mesh		100 Mesh		Pan	
	K.F. ^a	D.B. ^b	K.F. ^a	D.B. ^b	K.F. ^a	D.B. ^b	K.F. ^a	D.B. ^b	K.F. ^a	D.B. ^b	K.F. ^a	D.B. ^b
0	8.52	9.31	8.52	9.31	8.52	9.31	8.52	9.31	8.52	9.31	8.53	9.31
2.5	8.50	9.29	7.20	7.76	6.25	6.67	5.45	5.76	4.63	4.85	4.03	4.20
5	8.27	9.02	6.53	6.98	5.84	6.21	4.92	5.17	4.29	4.48	4.18	4.36
15	5.64	5.98	4.67	4.90	4.37	4.57	4.00	4.21	3.94	4.10	3.86	4.01
30	5.26	5.55	4.15	4.33	4.05	4.22	3.88	4.04	3.94	4.10	3.85	4.00
45	5.15	5.43	4.11	4.29	4.05	4.22	4.15	4.33	4.15	4.33	3.89	4.05

^a K.F. = Karl Fischer moisture value. ^b D.B. = 100 × g of moisture/g of dry weight.

Table II—Particle-Size Distribution in the Top Zones of the Basket and Moisture Contents of Granulation Dried in a Fluid Bed Drier

Screen Fraction	Percent on Screen			Moisture Content, %		
	0–7.6 cm ^a	7.6–15.4 cm	>15.4 cm	0–7.6 cm	7.6–15.4 cm	>15.4 cm
+20	13	12	8	4.5	4.8	4.8
-20/+40	4	4	6	4.3	4.1	4.0
-40/+60	19	21	21	3.8	3.9	3.9
-60/+80	29	30	30	3.8	3.9	3.8
-80/+100	14	15	15	3.8	3.9	3.8
-100/pan	21	18	21	3.8	3.9	3.9

^a Zone of basket.

dix) that the drying curve be of the form:

$$\log(\text{RH} - \text{RH}_0) = (\alpha'/2.3)t + \beta' \quad (\text{Eq. 16})$$

where RH is the relative humidity in the outlet air, RH₀ is the relative humidity of the inlet air, and α' is independent of air velocity. As seen in Fig. 3, α (Eq. 12b) is dependent on air velocity and the rate-limiting step here cannot be moisture movement within the granule.

SUMMARY

Fluid bed drying is governed by external diffusion because:

1. Equation 12b predicts a found dependency on interstitial velocity but no temperature dependence. The lack of temperature dependence is shown in Fig. 2.
2. Data demonstrate a dependence on particle size predicted by external diffusion (Eq. 15) but not internal diffusion (Eq. A7).
3. The dependence of drying on velocity is identical to that expected from hydrodynamic film theory.

With dried granules, the larger the moisture content the larger is the granule. The granules do not equilibrate on storage.

APPENDIX

If, as shown schematically in Fig. 1, the air flow is Q cm³ of dry air/sec and the dry bulb temperature, T, of the stream is constant, and if the air contains, on entry, q₀ g of water/cm³ of dry air, then

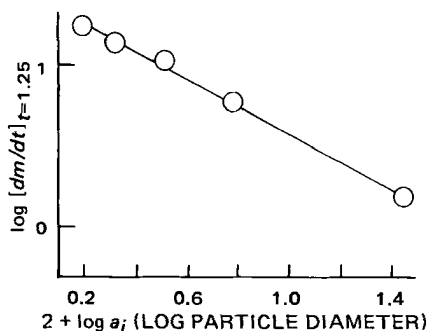


Figure 4—Negative slope estimated from Table I at time t = 1.25 min, plotted logarithmically versus the logarithm of the mean diameter of the screen cut.

the relative humidity (RH) of the incoming air will be (q₀/q*)100, where q* denotes the saturation amount of moisture per cubic centimeter of dry air at T₀. The air mass “gains” -dm/dt g of water/sec, where m is the moisture content of the granulation and where m₀ is the initial moisture content of the granulation. In time element dt, therefore, Q dt cm³ of air gain (-dm) g of water, so that,

$$\text{RH}/100 = [q_0 - (dm/Q dt)]/q^* = (\text{RH}_0/100) - (1/Qq^*) dm/dt \quad (\text{Eq. A1})$$

Pitkin and Carstensen (5) showed that for a monodisperse fraction of a granulation:

$$m = m_0(6/\pi^2) \sum_{j=1}^{\infty} (1/j^2) \exp(-j^2t/k) \quad (\text{Eq. A2})$$

where k = a²/(4π²D).

As before, if there is a fraction f₁ of particles with diameter a₁, f₂ with diameter a₂, etc., then:

$$m = \sum_{k=1}^{\infty} \sum_{j=1}^{\infty} f_k m_0(6/\pi^2)(1/j^2) \exp[-(j^2 4\pi^2 Dt)/a_k^2] \quad (\text{Eq. A3})$$

Differentiating yields:

$$-dm/dt = -m_0(6/\pi^2) \sum_{k=1}^{\infty} \sum_{j=1}^{\infty} f_k 4\pi^2 D/(a_k^2) \times \exp(-j^2 4\pi^2 Dt/a_k^2) \quad (\text{Eq. A4})$$

This equation, inserted in Eq. A1, yields:

$$(\text{RH} - \text{RH}_0)/100 = (m_0/Qq^*) \sum_{k=1}^{\infty} \sum_{j=1}^{\infty} f_k 24D/(a_k^2) \times \exp(-j^2 \pi^2 Dt/a_k^2) \quad (\text{Eq. A5})$$

Table III—Particle-Size Distribution versus Time of Drying

Screen Fraction	5 min	15 min	30 min	45 min
+20	67, 60	59, 60	60, 60	58, 59
-20/+40	17, 18	16, 16	17, 17	18, 17
-40/+60	10, 13	14, 14	14, 14	15, 14
-60/+80	3, 4	5, 5	4, 4	5, 5
-80/+100	1, 2	2, 2	1.5, 1.5	1.5, 1.5
-100/pan	2, 3	4, 3	3.5, 3.5	3.5, 3.5

Table IV—Screen Analysis and Moisture Contents of Drum-Stored Granulation

Screen Fraction	Percent (w/w)				Moisture Content, %			
	Initial	2 Days	3 Days	5 Days	Initial	2 Days	3 Days	5 Days
+20	59	57	57	57	8.9	8.5	8.4	8.2
-20/+40	12	13	12	12	6.9	7.7	6.9	7.4
-40/+60	10	15	13	19	6.4	6.6	5.7	6.6
-60/+80	7	10	12	9	5.4	5.6	5.8	5.8
-80/+100	4	3	4	1	5.0	4.1	4.4	4.0
-100/pan	8	2	2	2	4.5	3.6	4.1	3.9

If there is a predominance of a fine fraction (size a_1 , fraction f_1) where $j = 1$ and $k = 1$, then at reasonably high t Eq. A5 has one predominant term. It then simplifies to:

$$(RH - RH_0)/100 = (f_1 m_0 / Qq^*) (24D/a_1^2) \exp(-4\pi^2 Dt/a_1^2) \quad (\text{Eq. A6})$$

and hence:

$$\log(RH - RH_0) = -[4\pi^2 D / (2.3a_1^2)]t + \log(100C_0 24D) / (Qq^* a_1^2) f_1 \quad (\text{Eq. A7})$$

RH_0 is the relative humidity of the incoming air (not RH at time zero). According to Eq. A7, $t \rightarrow \infty$ implies that $RH \rightarrow RH_0$ in conformity with the physical picture presented. Hence, at conditions other than times strictly close to time zero, Eq. A7 predicts that the log of the term $(RH - RH_0)$ is linear with time. Figure 2 shows this to be so. However, the slope in Eq. A7 is independent of air velocity.

The efficiency of the dryer is calculated conventionally (6) as follows. Five kilograms of 10.22% initial moisture and 4% final moisture loses $511 - 187 = 324$ g water, so that $324 \times 0.582 = 199$ kcal is employed for evaporation. The bed in the process is heated 5° , i.e., $0.45 \times 4.489 \times 5 = 10$ kcal is used. Drying time is 15 min. The amount of heat given up is enthalpy of air at entry minus enthalpy at exit. The entry air is 43° , 10% RH; the exit air is 30° and the RH (Fig. 2) is estimated as follows. From Fig. 2 (slope A), $RH(t)$ from 0 to 15 min was measured and found to be 516 RH-minutes so that the average RH is:

$$RH = \frac{1}{15} \int_0^{15} RH(t) dt = \frac{516}{15} = 35\% \text{ FH} \quad (\text{Eq. A8})$$

The enthalpy of the outgoing air is found from a psychrometric chart to be $13 + 0.35(43 - 13) = 23.5$ Btu/lb dry air, and the enthalpy of the incoming air is $18 + 0.10(82 - 18) = 24.4$ Btu/lb dry air so the difference is 0.9 Btu/lb dry air. One pound of dry air at 110°F is found from the psychrometric chart to occupy 14.5 ft^3 , so the enthalpy difference is $0.9/14.5 = 0.062$ Btu/ ft^3 . The superficial

air flow is 2500 ft/min through a 0.166-ft^2 duct for 115 min; i.e., $2500 \times 0.166 \times 15 = 6225 \text{ ft}^3$. Hence, the heat expended is $6225 \times 0.062 = 386$ Btu = $386 \times 0.817 = 315$ kcal, so the efficiency of the dryer is $(199 + 10) 0.100/315 = 64\%$.

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